# ACTIVE STIFFNESS CONTROL OF A MANIPULATOR IN CARTESIAN COORDINATES[1]

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## ABSTRACT

A method of actively controlling the apparent stiffness of a manipulator end effector is presented. The approach allows the programmer to spécify the three translational and three rotational stiffness of a frame located arbitrarily in hand coordinates. Control of the nominal position of the hand then permits simultaneous position and force control. Stiffness may be changed under program control to match varying task requirements. A rapid servo algorithm is made possible by transformation of the problem into joint space at run time. Applications examples are given.

## INTRODUCTION

As manipulator use has moved from simple pick and place operations to more difficult assembly tasks the need for force control has become increasingly apparent. Properly applied force control can reduce the positioning accuracy necessary to perform a given task and in fact make possible assembly tasks which would be otherwise impossible. The difficulty in knowing and controlling precisely the positions of objects being assembled extends beyond the problem of arm control. Variation in part locations in feeders, slip in the gripper, part tolerance and other effects introduce random variation in final positioning. Tight fitting parts and complex geometries will tend to jam or get stuck if the forces during assembly are not properly controlled [1]. Often it is the inherent compliance in an assembly situation that allows it to proceed at all. Gripper elasticity, servo gain and structural stiffness all contribute to the intrinsic compliance in a task. Intentional use of mechanical compliances has found good success in specific assembly situations[2]. We present here a general method of controlling the compliant behavior of a manipulator.

At least two issues are of current interest in manipulator force control: 1) the formulation of command structures which permit specification of desired positions and forces in terms of task requirements and 2) the use of sensors to actively increase the fidelity and dynamic range of force control.

Previous approaches to force control structures have been presented by Craig, Geschke, Shimano, Paul and Whitney among others. These approaches have been aimed at providing the programmer with a means of specifying and controlling forces and positions in a non-conflicting way. This usually involves specification of a set of position controled axes and an orthogonal set of force controlled axes. Shimano and Paul start with a partitioning of cartesian space and then find the best joint or joints to force servo to approximate the desired force and position commands. Craig goes further and involves all joints in satisfying the the cartesian position and force commands simultaneously. A further discussion of this partitioning into natural and artificial constraints can be found in [3,4]. Geschke [5] takes a different approach by allowing somewhat arbitrary relationships between sensors and actuators to be established. For example he demonstrates a way to turn a crank by commanding a net force at the hand as a function of sensed position. The admittance matrix approach taken by Whitney [6] establishes a connection between sensed forces and commanded velocities by using a special matrix for each task. It is actually a velocity control scheme but has the net effect of controlling contact forces.

Active force control refers to the use of force sensing elements (usually strain gauges) to sense and provide for correction of force errors. The sensors may be placed at the actuator [7], at the wrist [3], at the fingers [8] or in the environment which the manipulator is contacting.

The stiffness control method described here provides a somewhat intuitive format for simultaneous motion and force command. It allows the programmer to think in terms of desired positions of objects. The stiffness value in one sense represents the accuracy which one expects to be able to satisfy position commands. For example if we expect to meet some physical constraint in a particular direction the stiffness in that direction is made low to insure low contact force and minimize the resulting friction forces. Conversely in directions in which we do not expect to meet physical constraint the stiffness is made high so the hand will follow closely the desired position. This allows us to resolve discrepancies between desired and achievable positions without excessive contact forces.

## BASIC STIFFNESS FORMULATION

By stiffness we refer to the rate at which forces and torques on the hand (hereafter called forces collectively) increase as it is deflected from a nominal position. In

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the following development we seek to transform a stiffness specification made in cartesian hand coordinates into joint coordinates. The purpose is to simplify the control computation while retaining the generality of task related stiffness specification.

The basic stiffness formulation follows from a generalization of the linear spring relationship, $f = kdx$, to a six-dimensional matrix expression,

$$F = K\delta X. \tag{1}$$

where $\delta X$ is a generalized displacement from a nominally commanded position, $X_0$, of the hand origin. We desire to produce restoring forces or torques along each axis of motion proportional to the displacement away from the nominal position. In order to preserve the linear character of the formulation we define $\delta X$ to consist of three orthogonal translation components and three small rotations about orthogonal axes. In practice this assumption has shown to be not severely restrictive.

Using the differential transform or so called jacobian matrix, $J$, we can determine cartesian displacement from the joint angle displacements. Defining $\delta\Theta = \Theta - \Theta_0$ as the difference between the actual joint angles and the nominally commanded joint angles we have

$$\delta X = J\delta\Theta. \tag{2}$$

Assuming the static and dynamic forces are compensated for or small enough to be neglected we can compute the joint torque, $T$, necessary to apply a force, $F$, at the hand. The jacobian transpose matrix establishes the relationship

$$T = J^T F. \tag{3}$$

Combining eq (1) thru (3) we arrive at an expression for joint torques necessary to make the hand behave as a six-dimensional spring in cartesian space

$$T = JKJ^T\delta\Theta. \tag{4}$$

The term $K_\Theta = JKJ^T$ is called the joint stiffness matrix. It should be noted that the jacobian in eq.(4) may be computed for any point fixed in the hand frame of reference. We thus are able to fix the stiffness center at an arbitrary position and orientation relative to the hand.

This formulation is central to the stiffness control method. It has the advantage that it requires only computation of the jacobian matrix and not its inverse. The jacobian itself is relatively simple to compute and can be evaluated rapidly from knowledge of intermediate joint frame locations [9]. $K_\Theta$ is decidedly non-diagonal (though symetric). This means that position errors in one joint will affect the commanded torque in all the other joints; the joint stiffness are highly coupled with each other. It is this coupling which allows us control over the location of the stiffness center and cartesian stiffness in hand coordinates. By stiffness center we mean the point at which pure forces may be applied and cause only translation of the hand. Alternately is is the point thru which all resulting rotation axes will pass if an arbitrary pure torque is applied to the hand.

## CONTROL MODEL AND IMPLEMENTATION

In order to make the arm behave with the desired stiffness described above we must, at the innermost level, be able to apply controlled forces at the hand. It has been found useful to be able to superimpose bias forces, $F_B$, on the stiffness behavior described above so that the resulting joint torque command is

$$T_C = K_\Theta\delta\Theta + T_B \tag{5}$$

where $T_B = J^T F_B$. Among other things this allows us to apply position independent forces at the hand by setting the stiffness in (about) the desired direction to zero.

While it is not necessary to use closed loop (active) force control with the stiffness approach, without it the sensitivity of force application is severely reduced. By taking force readings with the wrist sensor, $F_S$, we may determine the torques, $T_S$, on the individual joints by again using the jacobian matrix

$$T_S = J^T F_S. \tag{6}$$

This information is then used to determine the torque error on each joint, $\delta T = T_C - T_S$, and allow us to correct the applied motor torques so that the desired contact force is maintained at the hand. As our force sensor is placed close to the point of interest it is an appropriate estimate of contact forces. Prior to any contact with the environment a zero reading is taken on the force sensor so that the object weight will not add to the commanded forces.

The torque applied to the $i$th joint is given by the expression

$$T_i = T_{C,i} + G_i\delta T_i + K_{V,i}C_{II,i}\delta\dot{\Theta}_i + V_{0,i}sgn(\dot{\Theta}_i) + C_{I,i} \tag{7}$$

where:

$T_{C,i}$ =commanded torque, $i$th joint
$\delta T_i$ =torque error, $i$th joint
$\dot{\Theta}_i$ =velocity, $i$th joint
$\delta\dot{\Theta}_i$ =velocity error, $i$th joint
$G_i$=torque compensation function, $i$th joint
$K_{V,i}$ =velocity damping term, $i$th joint
$C_{II,i}$ =instantaneous inertia, $i$th joint
$C_{I,i}$ =gravity loading, $i$th joint
$V_{0,i}$ =friction torque, $i$th joint.

When the arm in contact with the environment we treat the dynamics as a lightly damped second order oscillatory system. Open loop impact tests were made while reading the force sensor at 1000hz to determine structural resonances. Depending on arm configuration resonant frequencies of 20 to 40hz were observed with contact occuring between a hard environment (aluminum table) and the hard edge of the fingertips. The natural frequency of the arm in contact with the environment depends on the elasticity of the environment and the arm itself and upon the distributed mass of the arm. As such it is difficult to model effectively and the force compensation gains were set for the worst case contact situation of the arm holding a rigid object in contact with rigid environment (typical in assembly situations). A zero order hold approximation of a lead-lag filter was used to stabilise the force feedback. Prior to converting the strain gauge readings to digital representation they are passed thru a low-pass analog filter to prevent aliasing of high frequencies in the system. To reduce steady state force errors an integrator is placed in parallel with the lead-lag compensation. It is preceded by a deadband and limiting non-linearity. The deadband tends to reduce limit cycling and the limiter tends to reduce the effective gain of the integrator for large force errors (such as impact transients, which we wish to ignore).

For each joint gravity loading and instantaneous inertia are calculated using Bejczy's approach [10]. While the intended operation of this system is in contact with the environment there are situations where we operate out of contact with it. In this case the force sensor behaves as an accelerometer with gain depending on the mass in the hand. If the mass in the hand is small this acceleration feedback does little to damp the motion. It is therefore necessary to add velocity dependant damping. This is done on a joint by joint basis. For stability with changing inertias we multiply the velocity feedback term for each joint by the inertia of the joint. We expect to introduce ultimately damping in cartesian space as a function of cartesian stiffness values which will be transformed to joint space in the same way the stiffness specification is.

The most severe non-linearity in the force control loop is the Coulomb friction in the joints. By closing the control loop around it we endeavor to minimise its effect on the system. Ideally we would place the sensor just after the gear reduction unit [7]. By placing the sensor close to the actuator we would minimize the dynamic complications of varying arm inertia and elasticity on the controllability of this state. On the other hand we would be unable to observe the effects of imperfectly modeled gravity loading on the contact forces at the hand. Ultimately we may find that some combination of joint and hand sensors will yield the best results in high performance force control. For the instant we deal with a manipulator with a wrist mounted force sensor only. A block diagram of the control system appears in Figure 1.

This system is implemented in assembly language on a PDP 11/45 with cash memory. It requires 5.3 msec to compute the drive values for all 6 joints of the Scheinman-Stanford arm. The process is run at 60 Hz so that two arms may be servoed at once. A background job runs at 12 hz to update values of $J$ and $K_\Theta$. The nominal position and velocity values ($\Theta_0 and \dot{\Theta}_0$) are generated by interpolation of a 4th order polynomial. The high level trajectory calculator generates spline curves in joint space that will drive the nominal position of the hand thru points defined by the AL programmer. In the next section we will describe the AL language constructs which allow motion specifications to be made with stiffness control.

## HIGH LEVEL CONSTRUCTS

In order to use the above described force capabilities we have added several new commands to the AL language. Motions are commanded with the MOVE statement. Clauses may be added to MOVE statements to introduce positions ("VIA" points) thru which the arm must move, to test for various conditions and otherwise modify the motion. A more detailed explanation may be found in the AL users manual [11]. The new clauses added for stiffness control are 1) "WITH STIFFNESS=(Kx,Ky,Kz,Gx,Gy,Gz) ABOUT trans", 2) "WITH FORCE (unitvector) = value", and 3) "WITH TORQUE (unitvector)

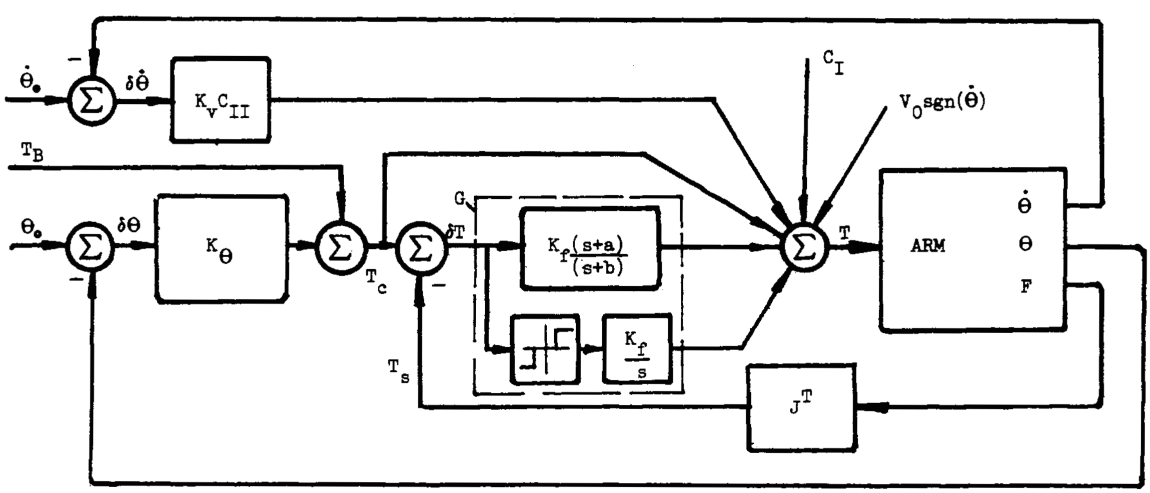


Figure 1. Stiffness Control System.

= value". The first clause sets the 3 translational and 3 rotational stiffnesses of the controlled frame. Kx,Ky and Kz are given in oz/in and Gx,Gy and Gz are in oz-in/rad. The controlled frame is located at a position and orientation fixed relative to the hand by "trans". The last two clauses permit bias forces of magnitude "value" to be added to the motion in the direction of "unitvector" which is one of the principal directions in the stiffness frame. Forces are in oz and torques in oz-in. The new clause "WITH GATHER = (fx,..., mx,...) allows force and torque readings to be stored during the move for later graphing. This information can then be viewed immediately after the move with a graphics and AL control program developed by Goldman [unpublished]. This very convenient system allows one to evaluate rapidly the operation of a force strategy. The sensitivity of the existing force testing statements such as "ON FORCE(zhat)≥ 10 oz DO ..." has been increased to the level of several ounces by using the wrist sensor. Formerly force sensing clauses tested only motor drive values with a resulting poor sensitivity of no better that 80 oz.

## AN EXAMPLE

As part of a demonstration of new manipulation capabilities an assembly problem was undertaken. A garden sprinkler shown in Figure 2 was selected because the geometry presented a variety of difficult assembly problems that appeared to be solvable with the stiffness control approach.

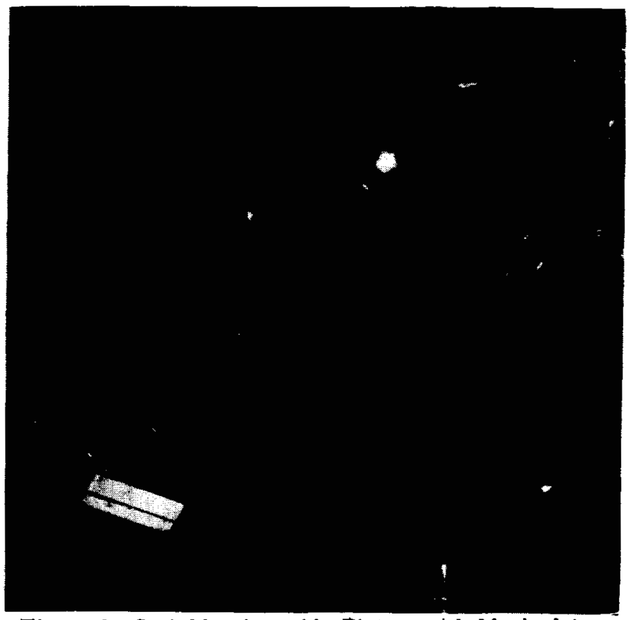

Figure 2. Sprinkler Assembly Fixture with Manipulator Hand and Force Sensing Wrist.

The final assembly step is the insertion of the sprinkler stem thru a hole in the sprinkler head and then thru an o-ring which is supported from behind by a fixture in the assembly station. Figure 3 shows the situation schematically and indicates the coordinate axes used.

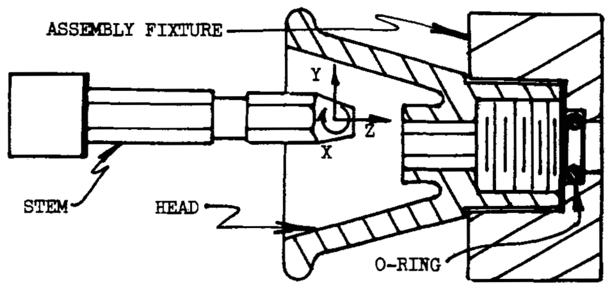


Figure 3. Cutaway View of Sprinkler in Assembly Fixture.

The stem has a hexagonal cross section as does the hole into which it fits. The lack of precision possible in determining this position requires a relatively requires a low rotational stiffness about the stem axis (z-axis). The hole is in a flat surface located at the bottom of a well in the sprinkler head. After positioning the stem in the hand and over the head in the assembly station the hand moves in the direction of surface until contact is detected from force readings. Under stiffness control the hand then made to move along a line parallel to the surface where the hole is located (along y-axis). A bias force normal to the surface is added to insure contact with the surface and to cause entry into the hole when the stem tip crosses it. Positions are unconstrained along the line of desired motion so we make the stiffness in that direction relatively large. Random positioning errors make finding the exact location of the hole impossible and we must find it thru some strategy. By making the third translational stiffness (x-axis) low we allow the stem tip to naturally center itself on the curvature of the well wall as it is driven into it. Thus upon moving back in the -y direction the stem will tend to be centered over the hole and are bound to enter it as we pass over it. The rotational stiffness about the x and y-axes are kept high to preserve the orientation in these unconstrained directions. To determine if the hole was entered the insertion depth is measured by reading the arm position. If the depth is not large enough the process is repeated until the tip enters the hole. At this point we are assured of being started into the hole. To ensure that binding will not occur once inside the hexagonal hole all the rotational stiffness are made low. As additional assurance against jamming the stiffness center is placed at the tool tip. A sequence of pushes and twists was found experimentally that reliably completed the assembly. We start with a bias force and increase the insertion force in three steps by attempting to push the stem further into the hole. Simultaneously we twist back and forth about the stem axis and try to translate back and forth along the y-axis. This tends to help the stem move past a sharp shoulder that is machined into it. This process may take several repetitions to ensure success. Ultimately the stem moves far enough to enter the o-ring. During the operation force,time and insertion depth are monitored. If sufficient insertion depth is attained the process is stopped and the completed part removed from the assembly station. The AL code for this process is:

```
down1←TRANS(NILROT,ZHAT);          {define relative motions }
down2←TRANS(NILROT,2*ZHAT);        {       for hand to make }
down4←TRANS(NILROT,4*ZHAT);        {       during insertion }
rotate30←TRANS(ROT(ZHAT,30),NILVECT);
rotatem30←TRANS(ROT(ZHAT,-30),NILVECT);
ywig←TRANS(NILROT,YHAT)
mywig←TRANS(NILROT,-YHAT)

stem_final← ... data...            {expected final positon  }
stem_tip←TRANS(NILROT,4*ZHAT);     {insertion stiff. center }
stem_touch←barm;                   {record current position }

 {Repeat following MOVE until stem is inserted 1.85 in   }

DO BEGIN
 MOVE barm TO stem_final*down4     {try to push 4 in. beyond}
  DIRECTLY                         {        final position  }
  VIA stem_touch*rotate30*down1    {twist and push down     }
  VIA stem_touch*down2
  VIA stem_touch*rotatem30*down2
  VIA stem_final*down1*ywig        {push down harder and
  VIA stem_final*down1             {        wiggle in y dir. }
  VIA stem_final*down1*mywig
  WITH FORCE_WRIST NOT ZEROED      {don't zero wrist datum  }
  WITH STIFFNESS = (30,30,50,50,40,40) {set stiffness       }
        ABOUT stem_tip             {with center at stem tip }
  WITH FORCE(ZHAT) = 15            {start with bias force   }
  WITH GATHER = (FX,FY,FZ,MZ)      {collect force histories }
  ON FORCE(ZHAT) ≥ 80 DO           {stop if force becomes
        STOP BARM                  {        too large       }
  WITH DURATION = 2.5*SECONDS;     {allow 2.5 seconds       }
                                   {compute insertion depth }
  depth←(POS(stem_touch)-POS(barm)).(ZHAT WRT stem_final);
END UNTIL depth ≥ 1.85;
```

Figure 4 shows the time histories of forces sampled at 60 Hz during the final insertion phase.

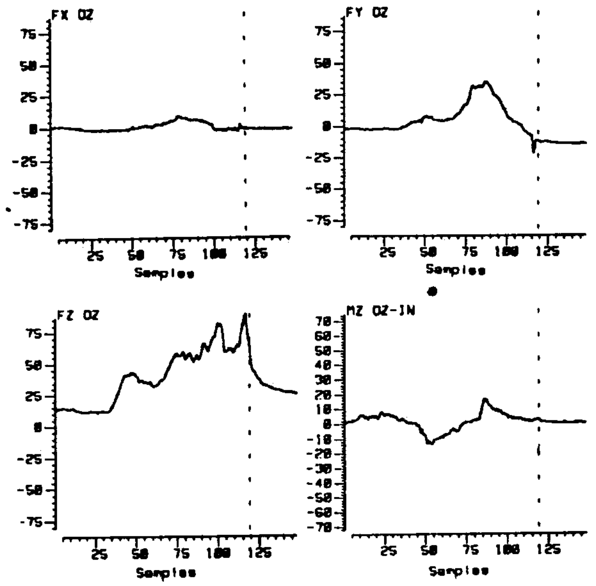


Figure 4. Force Histories During Final Insertion. Duration is 2.4 seconds at 60 Hz sample rate. Vertical dashed line indicates time when servos were turned off and brakes set.

The bias and stepped increase in the z-axis force (insertion force) can be clearly seen in the FZ plot. Around sample 80 the shoulder on the stem broke free of an obstruction. The z-force level falls temporarily while the stem slides forward and then contacts the o-ring. The sinusoidal y-force and z-moment can also be observed. Except for impacts the force changes are gradual. This results directly from the smooth position trajectories that the arm has been commanded to follow. When withdrawing the completed assembly from the fixture the stiffness center is moved above the part so that jamming does not occur as is it slides out of the assembly station. The ability to move the stiffness center to different locations under program control has proved to be a useful feature. It is particularly useful when the arm must handle a variety of different parts in the same assembly.

## APPLICATION TO OTHER DEVICES

Calculation of the joint stiffness term, $K_\Theta$, in equation 4 does not require that the jacobian matrix be square. In the case of a redundant manipulator with n>6 degrees of freedom this means that the stiffness formulation is still valid. $K_\Theta$ will be a n×n matrix and will still correctly allow us to compute the joint torques from joint displacements so that the hand exhibits the required 6 dimensional stiffness.

Similarly the approach may be used to control the behavior of an object grasped in a multi-finger articulted hand. We imagine a hand with three fingers, each with three degrees of freedom grasping an object with the fingertips. Six stiffness values can be used to establish the compliant behavior of the object relative to the hand and an additional three will be needed to control the internal forces between fingers transmitted through the grasped object.

## CONCLUSIONS

The stiffness control approach to force control in a manipulator system has been shown to be a useful and effective means of effecting force control in assembly tasks. Its formulation leads to a relatively straightforward control system. Commanded from a high level language such as AL, stiffness control allows the programmer to think in terms of desired part trajectories during assembly. With knowledge of expected constraint conditions in the assembly task the programmer is able to "shape" the stiffness specification to match the particular task and thereby prevent part jamming and undue friction forces during assembly. The addition of active force sensing has increased the sensitivity of force control under AL by almost an order of magnitude.

The encouraging results of this study will lead to experimental application to hand and multi arm situations.

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approach.

Figure 2. Sprinkler Assembly Fixture with Manipulator Hand and Force Sensing Wrist.